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ABSTRACT

Reported are results of a study designed to determine developmental patterns in proportional reasoning and to determine if these patterns are similar regardless of educational level and regardless of problem context. Data from 2282 subjects in grades 6-13 were collected using the Test of Logical Thinking (TOLT), and then examined for subjects who had been categorized in three ways: (1) by TOLT scores, 1-10; (2) by educational level; and (3) by developmental stages within each educational level. Examination of the four most common responses indicates that observed patterns of reasoning are similar regardless of developmental level, educational level, or problem context. Conclusions are made regarding the use of proportional reasoning in the physical sciences and biological sciences. (CS)

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PATTERNS OF REASONING: PROPORTIONAL REASONING

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Cognitive development has become an increasingly important area of study for educators and psychologists. While much of this interest simply reflects a need to understand the evolving thought processes of youngsters through adolescence, knowledge of cognitive development also has great potential in understanding the effectiveness of teaching and learning strategies. Science and mathematics educators, in particular, have cited formal reasoning ability as an important mediator of achievement (e.g., Cantu and Herron, 1978; Goodstein and Howe, 1978). Five reasoning modes have been identified as having particular relevance to science instruction: controlling variables, proportional reasoning, probabilistic reasoning, combinatorial reasoning, and correlational reasoning.

If allowance for variation in reasoning ability is to be made in instruction, then efforts must be made to understand these various reasoning modes and their development. Traditionally formal reasoning has been assessed in an individual interview with standard problem situations presented in a flexible question sequence based on models developed by Piaget and his colleagues. Numerous studies in recent years have reported the development of reasoning skills (e.g., Lawson and Wollman, 1976; Levine and Linn, 1977). Of the five formal modes, proportional reasoning has probably received the greatest attention in the literature. This emphasis underscores the important role that proportional reasoning has in the attainment and mastery of many mathematic and scientific skills and concepts. The planning of this study resulted from a realization of this important role and from a desire to explore the development of proportional reasoning in adolescents. Focusing on incorrect response patterns can

facilitate progress in understanding the cognitive base of proportional reasoning.

Purpose

Four questions were examined in this study.

1. Are there large portions of middle school, high school and college students who are not utilizing proportional reasoning to solve problems?
2. Is the mode of reasoning just prior to the development of a proportionality schema primarily an additive one?
3. Are the observed developmental patterns in proportional reasoning similar regardless of educational level? and
4. Are the observed developmental patterns in proportional reasoning similar regardless of problem context?

Procedures

Data from 2282 subjects in grades 6-13 were collected using the Test of Logical Thinking (TOLT). (Tobin and Capie, 1981) Approximately 1627 used Form A and the balance used Form B. Subjects taking Form A consisted of nearly equal numbers of middle school, high school, and college students while two-thirds of the subjects taking Form B were college students. The psychometric characteristics of TOLT have been well documented by their developers (Capie and Tobin, 1980). The item format and the existence of parallel forms made these test forms well-suited to this investigation.

In each TOLT form subjects are presented two problems requiring proportional reasoning. Form A problems involve making orange juice from oranges; Form B problems involve painting rooms with cans of paint. The ratio used in all problems is 3:2. In the first item on each form subjects must derive "9" given "6"; in the second item they must derive "8 2/3"

given "13". Thus, the two problems on each form differ quantitatively (whole number vs. fraction) and qualitatively (direction of operation). A sample TOLT item is given in Figure 1.

After reading the problem the subjects select an answer from five choices given and then they must identify their reason from among the five alternatives listed. Twenty-five combinations, then, are possible. The four most common combinations, which accounted for 87% of the responses, were selected for additional study.

The data were examined in three ways.

1. Subjects were sorted by TOLT scores, 0-10, to represent increasing formal reasoning ability.
2. Subjects were sorted into three educational levels (MS, HS, College).
3. Subjects were sorted into three developmental stages (concrete, transitional, formal) in each of the three educational levels.

The frequencies of the most common response combinations were then determined in each of these data analyses. Interpretation of the results was enhanced by the construction of a series of plots of frequency of response against category of interest (e.g., educational level).

A chi-squared analysis was used to determine if the response patterns for each of the two proportional reasoning tasks differed for concrete and transitional students across educational levels.

Results

The four common response combinations that occurred for each of the proportional reasoning problems, in order of frequency, were:

1. correct answer and correct reason;
2. focus on a single difference (additive thinking);

3. there is no way to predict;
4. correct reason but with an arithmetically incorrect answer (e.g., 9 instead of 8 2/3).

Subjects experienced greater success on the first proportional reasoning items than on the second items. Table 1 gives the weighted averages (items from each TOLT form combined) for the percentage of respondents selecting the correct answer and reason.

The analysis of the data sorted by TOLT scores 0-10 was conducted by constructing the graphs previously described. A sample plot for item one on Form A is given in Figure 2. The plots for the other three items were quite similar to the one shown.

Table 2 displays data for students of different cognitive levels within each educational level. The developmental categories of interest were concrete (TOLT scores 0-1) and transitional (TOLT scores 2-3). Non-significant chi-squared values showed that response patterns for concrete and transitional subjects were similar for middle school and high school students. The values obtained were $\chi^2_{df = 3} = 7.76$, $\chi^2_{df = 3} = 4.64$, $\chi^2_{df = 3} = 0.74$ and $\chi^2_{df = 4} = 1.33$, respectively.

Discussion

An examination of responses by educational level (see Table 1) reveals that there are indeed large numbers of students who could not solve problems requiring proportional reasoning. On the more difficult second items, these percentages are particularly alarming. While the numbers do show an expected increase across educational level corresponding to increasing TOLT scores, they also reveal that the college students had particular trouble with these second items. These results certainly underscore the findings of other investigations (e.g., Karplus et al., 1977) that large numbers of adolescents have not attained proportional reasoning.

The results also reveal that the incorrect responses that students make are not haphazard; they clearly indicate common errant modes of reasoning. The most common incorrect response was one of additive thinking, as suggested by others (e.g., Inhelder and Piaget, 1958; Karplus and Karplus, 1972). This pattern of reasoning involves a focus by the student on only a single difference in the objects. Reasons three and four (see Figure 1) belong to this category. The second most common incorrect response was that "there is no way to predict." The frequency of this response was greatest among college students. Perhaps college students' experience with standardized and similar testing prods them to find such answers attractive as though some "trick" were involved. Of course there may be students who mentally object to dividing oranges or cans of paint (required on second items only).

The third observed pattern of reasoning was that of choosing the correct reason (ratio) but with an arithmetically incorrect answer. This response was more common for the second problem, particularly among the college students. Since the second items involve fractions, these observed results are not unexpected. Surprisingly, however, this phenomenon was especially associated with the college students. One would be surprised indeed if pre-college students have better arithmetic skills than college students.

With the exceptions noted above, the observed response patterns are similar across educational level. The chi-squared analyses showed that response frequencies for concrete and transitional subjects are similar for middle school and high school students. Significant chi-squared values, on the other hand, were obtained when the college students were included in the analysis. These significant values result from the increased frequency of "there is no way to predict" by college students.

Problem context has been cited by Bady (1978) as a significant variable in assessing reasoning ability. Although limited in this study to two

world objects (oranges and paint), the two TOLT forms do provide some variance in context. The results suggest that the developmental patterns are similar for these two problem contexts.

The results of this study also point out some additional questions. For instance, why is the percentage of students correctly answering the second problem (Form A and B) substantially lower than those for the first problem? Since the two problems differ both quantitatively (whole numbers vs. fractions) and qualitatively (direction of operation), an additional analysis was needed. Consequently, subjects who correctly answered the first item were selected (N = 213) in order to examine their responses on the second item. If the source of variation is arithmetic then one would expect that large numbers of these students would choose the correct reason but with an arithmetically incorrect answer. Sixty-five percent of this group did indeed choose the correct reason. The results of this analysis suggest that the level of arithmetic computation is an important variable and one that needs to be given attention in future studies.

Finally, one might ask if the development of proportional reasoning is a gradual or abrupt phenomenon. Pallrand (1979), studying the same three educational levels, has found that this development seems to be gradual. Although this research question would ideally involve a different design, the results from the second problem in this study (see Figure 2) do seem to support the idea of continuous development. The results from the first items, however, reveal a curious dip between TOLT scores 3-5. Further studies of these transitional and early formal thinkers are needed.

Conclusions and Implications

The fact that a majority of students cannot solve problems requiring

proportional reasoning has serious implications for science instruction. The physical sciences, in particular, abound with concepts and laws that require proportional reasoning: density, gravitational force, gas laws, stoichiometry, etc. Biological science, too, deals in proportions when a topic involves overlap with physical science (e.g., energetics). And, understanding proportions is essential to a study of genetics.

Science teachers and educators should no longer assume that students enter the science classroom and laboratory equipped with the cognitive ability to handle these concepts. Awareness of the developmental patterns that exist can be beneficial. Knowledge by the teacher that concrete and transitional thinkers are focusing on a single difference can aid the teacher in addressing this problem. Study should also be made of the instructional strategies that have been shown to promote proportional reasoning (e.g., Wollman and Lawson, 1978). And as Cantu and Herron (1978) have pointed out, concrete thinkers not only have great difficulty with formal concepts, they do less well than their formal counterparts on concrete concepts.

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TABLE 1

Percentage of subjects correctly answering proportional items by educational level.

	Middle School	High School	College
Item One	15	56	77
Item Two	12	47	46

TABLE 2

Percentage of subjects selecting a common response by developmental level within educational level.

Item One	Concrete			Transitional			Formal		
	MS	HS	College	MS	HS	College	MS	HS	College
Correct answer and response	0	7	17	15	18	41	36	78	85
Focus on single difference	66	60	50	57	48	25	31	8	3
No way to predict	5	7	25	2	8	11	7	3	5

Item Two	Concrete			Transitional			Formal		
	MS	HS	College	MS	HS	College	MS	HS	College
Correct answer and response	3	4	8	8	10	8	28	67	54
Focus on single difference	38	35	17	43	36	14	19	5	2
No way to predict	5	7	25	5	4	16	12	6	9
Correct reason, arithmetically incorrect answer	0	4	0	5	6	24	1	5	22

House Paint #1

A painter uses four cans of paint to paint six rooms. How many rooms can be painted with six cans of paint?

a. 7 rooms
b. 8 rooms
c. 9 rooms
d. 10 rooms
e. other

Reason

1. The number of rooms compared to the number of cans will always be in the ratio of 3 to 2.

2. With more cans of paint, the difference will be less.

3. The difference in the numbers will always be two.

4. With four cans of paint the difference was 2. With six cans of paint the difference would be two more.

5. There is no way of predicting how much paint is needed.

Figure 1. TOLT Item One from Form B.

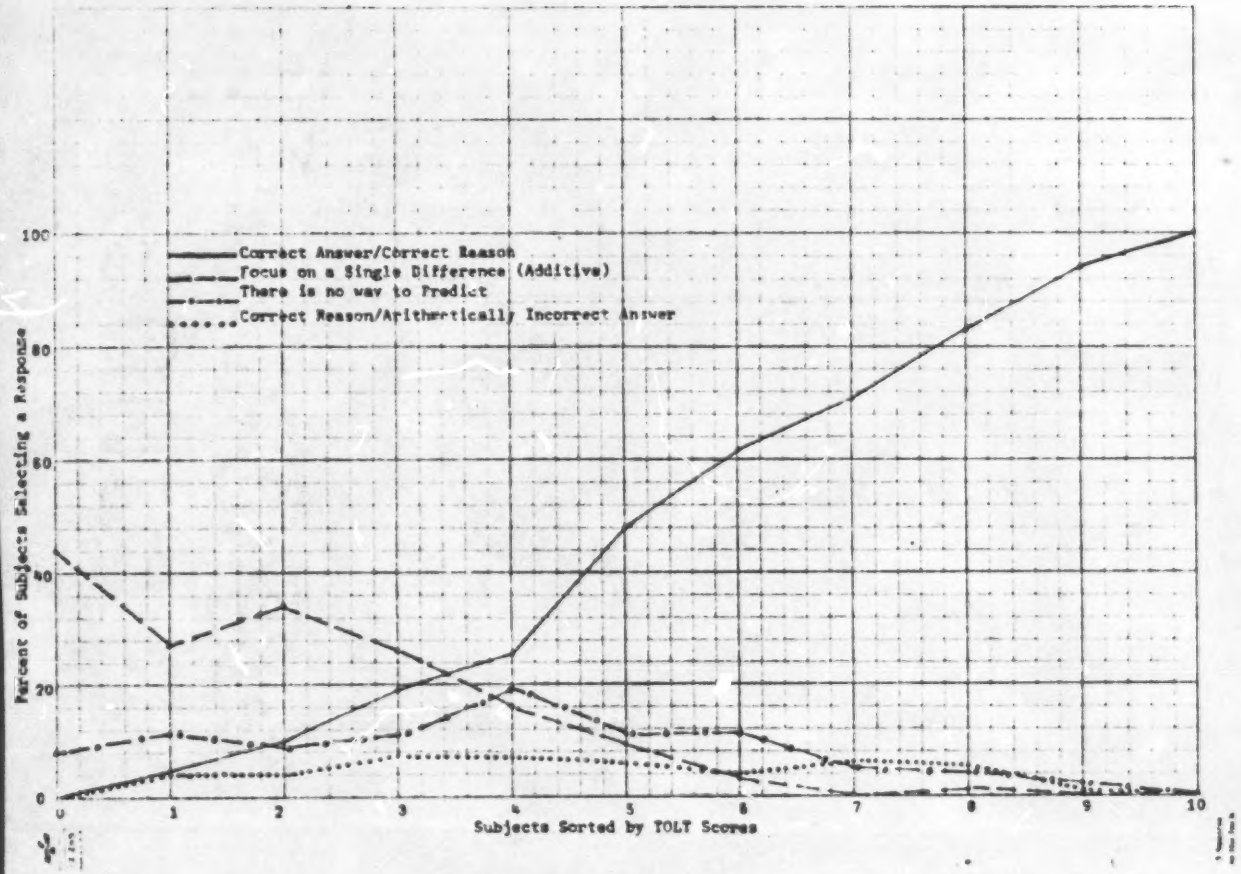

Figure 2. Plot of percent of subjects selecting a response against subjects sorted by TOLT scores 0-10.